Leaflet No.8/82-83

Government of the Union of Myanmar
Ministry of Forestry
Forest Department
Forest Research Institute

Log Volume Estimation by Using Spline Interpolation

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February 1983

(Spline Interpolation) နည်းကိုအသုံးပြု၍ သစ်ထုထည်ခန့်မှန်းခြင်း

ဦးထွန်းလင်း
သစ်တောသုတေသနဗိမာန်

စာတမ်းအကျဉ်းချုပ်

သစ်ပင်၏ ပင်စည်ဖွဲ့စည်းပုံကို (Spline) မှီချက်များ အသုံးပြုလေ့လာထားသော စာတမ်းဖြစ်ပါသည်။ ရရှိသည့်မှီချက်များသည် သစ်ထုထည်ခန့်မှန်းရာတွင် တိကျမှန်ကန်ကြောင်း တွေ့ရှိရသည့်အပြင် သစ်တောသယံဇာတ စာရင်းကောက်ယူရာတွင် (Spline) မှီချက်များ၏ အသုံးဝင်မှုကိုလည်း လေ့လာတင်ပြ ထားပါသည်။

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Abstract

Natural cubic spline interpolation was employed to portray stem-taper. Derived functions provide accurate results in log volume calculations. The feasibility of using spline approximations in forests inventories has also been examined.

Contents

1. Introduction

This study uses cubic spline functions to portray stem taper and to calculate by volumes. In forestry practice, stem taper refers to the decrease in size (e.g radius) of a tree's stem from the bottom upward. The intrinsic structure of spline functions has been treated throughly by Ahlberg et al. (1967) and Bickley (1968). Basic working equations are outlined in the next section.

Numerical approximation of stem taper is of great concern to forest mensurationists. Foresters have customarily used a single taper equation in estimating stem taper and, accordingly, the log volumes of trees of a wide range of species and geographical areas. This practice relies on the assumption that taper in all species is in accordance with the same fundamental growth principles, and that the main stem of a forest tree might conform to the dimensions of certain predignated geometric solids. Liu (1980). With this approach, researchers have optimistically hoped that a given sample tree would have a changing radius profile similar to that assumed by various taper functions. Contrary to this general view, Liu (1980) doubted that any preconceived functional form could represent stem taper properly. Based on the observation that individual trees seem capable of assuming an infinite variety of shapes, he asserted that an explicit analytic definition of stem taper lacked generality. Yet, he indicated that the task could be accomplished with a sampling approach restricted to a smaller population.

There is biological, analytical, and visual evidence that trees of the same species, growing in the same geographical area and receiving similar treatments, assume a characteristic shape. However, deviation from this prototypic sketch are common. Stem taper is an unstable factor sensitive to many interactions in a dynamic forest system. Therefore, fitting a curve to any preconceived functional form may fail to trace stem profile. A more rational approach is to provide a numerical technique that is capable of assuming various functional forms depending on the distribution of data points. Such a technique can be developed by the use of cubic spline approximation, as follows.

2. Methods

This study used cubic spline functions to portray stem taper with application of Reinsch's (1967) algorithm. Spline approximation is an interpolation by a class of coordinates functions which may be described as a set of cubic polynomial segments with smooth joints (Fig 1). Instead of approximating a given function f(x) over an internal [a,b] by a single polynomial, use may divide [a,b] into n subintervals $[a,x_1]$, $[x_1, x_2]$,……$[x_{n-1}, b]$ and approximate f(x) by a different polynomial on each subinterval.

In determining the approximating function g(x), it is required

(1) that in each subinterval the approximation function g(x) be a polynomial of maximum degree 3;

(2) that g(x) agree with f(x) at each of the (n +1) points $x_0 = a, x_1, x_2, \ldots\ldots, x_{n-1}, x_n = b$ and

(3) that the first derivative g' (x) and the second derivative g"(x) be continuos on [a,b].

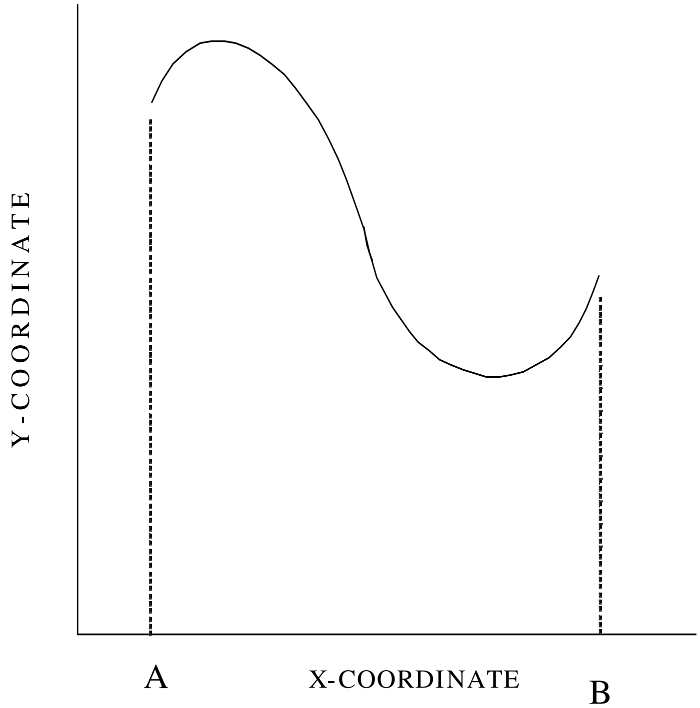

Fig 1. An Example of cubic spline

When (1), (2) and (3) are satisfied, g(x) is called a cubic spline function. In addition, the maximum curvature property of a spline function specifies that the mean square value of g''(x) over [a,b] is minimal. If g''(a) = 0 are selected as the auxiliary conditions completing the specification of g(x), then the approximation is called a natural cubic spline. Such conditions, generally imposed upon non periodic splines (Ahlberg et. al 1976), prescribe cantilevered ends for a cubic spline function.

Reinsch's algorithm furniches a spline function together with some kind of smoothing. It is formulated and optimal in a sense specified below.

Let x_i , y_i, i = 0,1, …. n be a given set of coordinates and assume that $x_0 < x_1 < x_2 \dots < x_n$

The smoothing function, $g \in c^2 [x_0, x_n]$ to be constructed shall minimize

$$\int_{x_0}^{x_n} [g^n(x)]^2 \, dx \qquad (1)$$

Among all function , g(x) such that

$$\sum_{i=0}^{n} \left(\frac{g(x) - y_I}{Wy_I} \right)^2 \leq S \qquad (2)$$

Here, $Wy_i > 0$, i = 0,1 …… n and $S \geq 0$ are given numbers

The constant S allows for an implicit rescaling of the quantities Wy_i which control the extent of smoothing. Appropriate value for S depend on the relative weights Wy_i^{-2}. If available, one should use for Wy_i an estimate of the standard deviation of the originate y_i . In this case, natural value of S lie within the confidence interval corresponding to the left – hand of (2);

$$(n-1) - (2n+2)^{1/2} \le S \le (n+1) + (2n+2)^{1/2} \qquad (3)$$

Choosing S equal to zero leads to the problems of interpolation by cubic spline functions.

Generally, log volume is calculated by the following formula;

$$v = \pi \int_{x_i}^{x_j} [g(x)]^2 \, dx, x_0 \le x_i \le x_J \le x_n \qquad (4)$$

where g(x) is a cubic spline approximation of individual tree's dimensional measurements of the form

$$g(x) = M_{j-1} \frac{(x_j - x)^3}{6h} + M_j \frac{(x - x_{j-1})^3}{6h} + \left(y_{j-1} - \frac{h^2}{6} M_{j-1} \right) \frac{(x_j - x)}{h} + \left(y_j - \frac{h^2}{6} M_j \right) \frac{(x - x_{j-1})}{h} \qquad (5)$$

Where $M_j = g''(x_j)$ for $x_j = x_o + jh$ (j = 0, 1, -------- n), If the approximation is based on relative radii and positional heights, then the log volume equation is of the following form:

$$v = \pi R^2 H \int_{x_i}^{x_j} [g(x)]^2 \, dx, a \le x_i \le x_J \le 1 \qquad (6)$$

Where R is a radius measurement and H is the total height of the tree.

3. Data Acquisition

The data of twenty one Ingyin (*Pentacme siamensis*) trees were randomly select from Shwebo forest inventory survey data (1971). Defective trees were eliminated during sampling. Each sample tree data was interpolated into fourteen radius at 0,2,4,6,8,10,20,30,40,50,60,70,80 and 90 percent of the total bole height by using Lagrange's formula

$$P(x) = \frac{(x - x_1)(x - x_2) \quad \ldots\ldots\ldots (x - x_n)}{(x_0 - x_1)(x_0 - x_2) = \ldots\ldots\ldots (x_0 - x_n)} y_o$$

$$+ \frac{(x - x_0)(x - x_2) \quad \ldots\ldots\ldots (x - x_n)}{(x_1 - x_0)(x_1 - x_2) \ldots\ldots\ldots (x_1 - x_n)} y_1$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$+ \frac{(x - x_0)(x - x_1) \quad \ldots\ldots\ldots (x - x_{n-1})}{(x_n - x_0)(x_n - x_1) \ldots\ldots\ldots (x_n - x_{n-1})} y_n \qquad (7)$$

To standardize radius measurements, i.e., to obtain relative sizes of cross sections at corresponding positional height, the radius of each section was divided by that of the basal disc. This procedure makes the data amenable to both the comparison of taper among trees and the interpretation of fitted spline functions, relative radii of sections were used as ordinates while their associated positional heights were treated as abscissas. The twenty one sample trees were divided into two groups. Data Group 1, consisting of 6 trees, was used for curve fitting. Trees in this group ranged in total bole height from 47.6 to 70 ft. and in dbhob (not used for curve fitting) from 2.12 to 2.94 ft. Their mean values were 59.1 ft. in height, and 2.55 ft. in dbhob. Data group 2, consisting of 15 sample trees, was used for testing the accuracy of spline functions in volume estimations. The height range of this subgroup is 24.6 to 72 ft. (mean 46.37). The dbhob range is 1.66 to 2.93 ft (mean 2.32 ft.). The partitioning of sample trees into subgroups was done in an arbitrary manner.

4. Applications and Results

The method of curve fitting, especially the method of finding a polynomial, is the method of undetermined coefficients. A given points, in curve fitting, gives information about a certain but implicit function that is to be approximated. The approximation is carried out by a certain numerical procedure, splines or least squares, that will describe the implicit function properly.

In this study, the implicit function, given by the set of coordinates corresponding average relative radii and positional heights of all sample trees in data group 1, defined the average taper curve of the data. It was first approximated by Reinsch's algorithm with an attempt at smoothing the taper curve. Values used for relative weights and the extent of smoothing were those recommended by Reinsch (1967) with S equal to the lower bound of equation (3). Figure 2 shows the original data points for each of the 6 sample trees. The relative radius at the tip of the tree was set to zero. Adjacent points were connected by straight lines. At each positional height, means and standard deviations of relative radii were also graphed. The fitted curve is plotted in figure 3 and apparently, the curve failed to trace the basal region curvature. It is therefore suggested that Reinsch's procedure or other smoothing procedures should not be used for the derivation of a taper curve but for the smoothing of isolated errors in the measured data set.

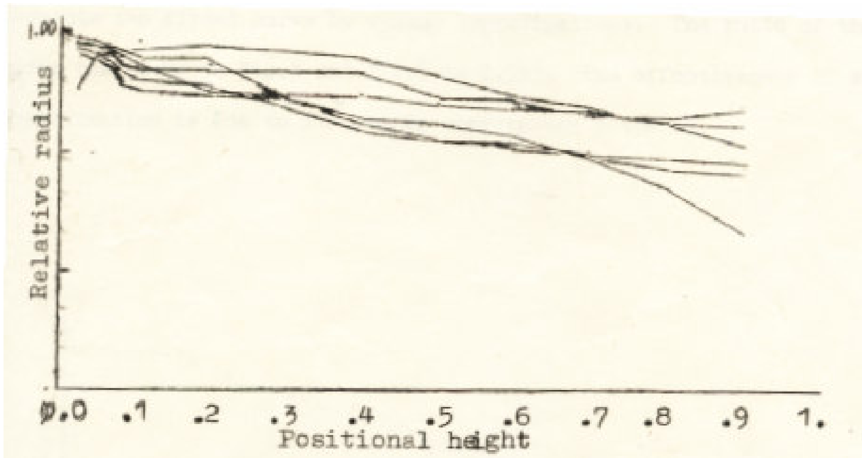

Figure 2. Stem profile of six sample trees in group 1

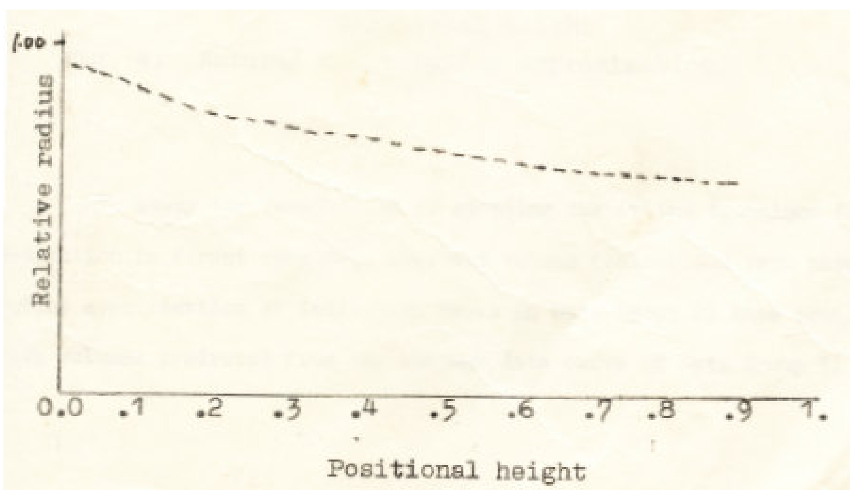

Figure 3. A balanced taper curve fitted by Reinsh's smoothing algorithm

Attention was then directed to the approximation of a natural cubic spline for the implicit function defined above. The resulting spline functions were evaluated at 51 points along the horizontal axis. Figure. 4 represents the fitted curve by spline approximations. The ratio of the areas under the curve in Figure.3 and Figure.4 is 0.983. The effectiveness of spline approximation is due to its strong convergence property.

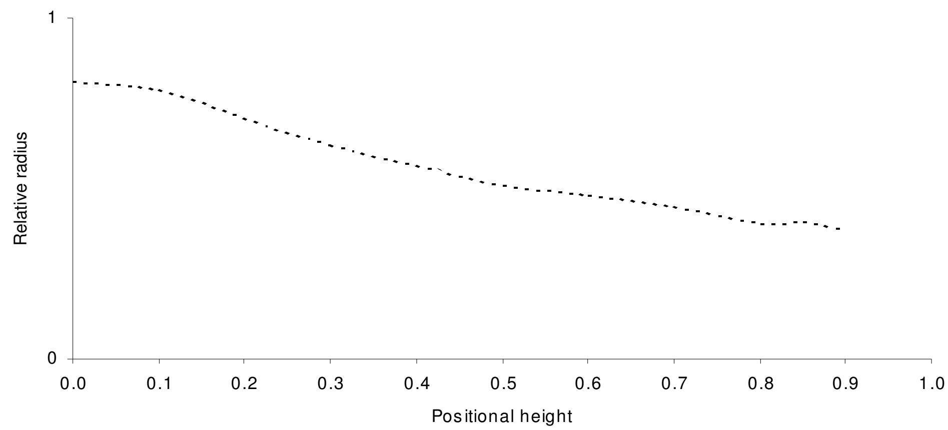

Fig 4. Natural Cubic spline approximation

To study the feasibility of adopting the spline technique for volume estimation in forest sampling, observed volume (calculated from natural cubic spline approximation of individual trees in data Group 2) have been compared with volumes predicted from the average data curve of data Group 1.

Table 1. Comparison of observed and predicted volumes (Data Group 2)

Tree No.	Dbhob (ft.)	Bole Height (ft.)	Stem Volume		Error
			Observed (cu.ft.)	Predicted (cu.ft.)	
1	1.66	40.6	40.83	41.25	-1.03
2	2.79	54.0	141.90	138.22	2.59
3	1.91	27.2	28.70	30.50	-6.27
4	2.64	37.7	72.89	70.66	3.06
5	1.97	29.0	40.48	39.42	2.62
6	1.88	24.6	31.58	30.22	4.31
7	1.17	47.6	65.79	66.28	-0.74
8	2.29	28.6	57.02	59.25	-3.91
9	2.83	61.0	156.42	149.21	4.60
10	2.93	65.2	162.90	158.54	2.67
11	1.72	40.0	48.41	49.62	-2.52
12	1.84	58.0	90.48	92.04	-1.72
13	2.51	62.0	108.93	105.65	3.01
14	2.71	48.0	94.56	93.22	1.41
15	2.93	72.0	150.11	145.21	3.26
Mean	2.32	46.37	86.07	84.62	.756

Table 1. shows comparisons of observed and predicted volumes made for the entire stem. Observed volume was based on equation (4) and dimensional measurements of individual trees in Data Group 2. Predicated volume was based on equation (6) and the measured basal radius and tree height of individuals in Data Group 2. The taper curve used for g(x) of Equation (6) was the cubic spline function derived from spline approximation of the average taper curve of Date Group 1. Estimation error in percent was calculated by (observed- predicted)/observed *100.

5. Discussion

The natural spline approximation technique is not only a powerful tool for stem taper analysis, it is also an accurate numerical procedure for log volume calculation. The method is particularly useful for those operations in which dimensional measurements of logs can be easily obtained. This method requires only a few sample trees to derive the taper function. To verify the usefulness of spline approximation approach, however, further studies on estimation of log volume for various species by spline functions should be continued.

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